Ocean Waves
Basic Theory and Best Practice as Applied to Buoy Measurements

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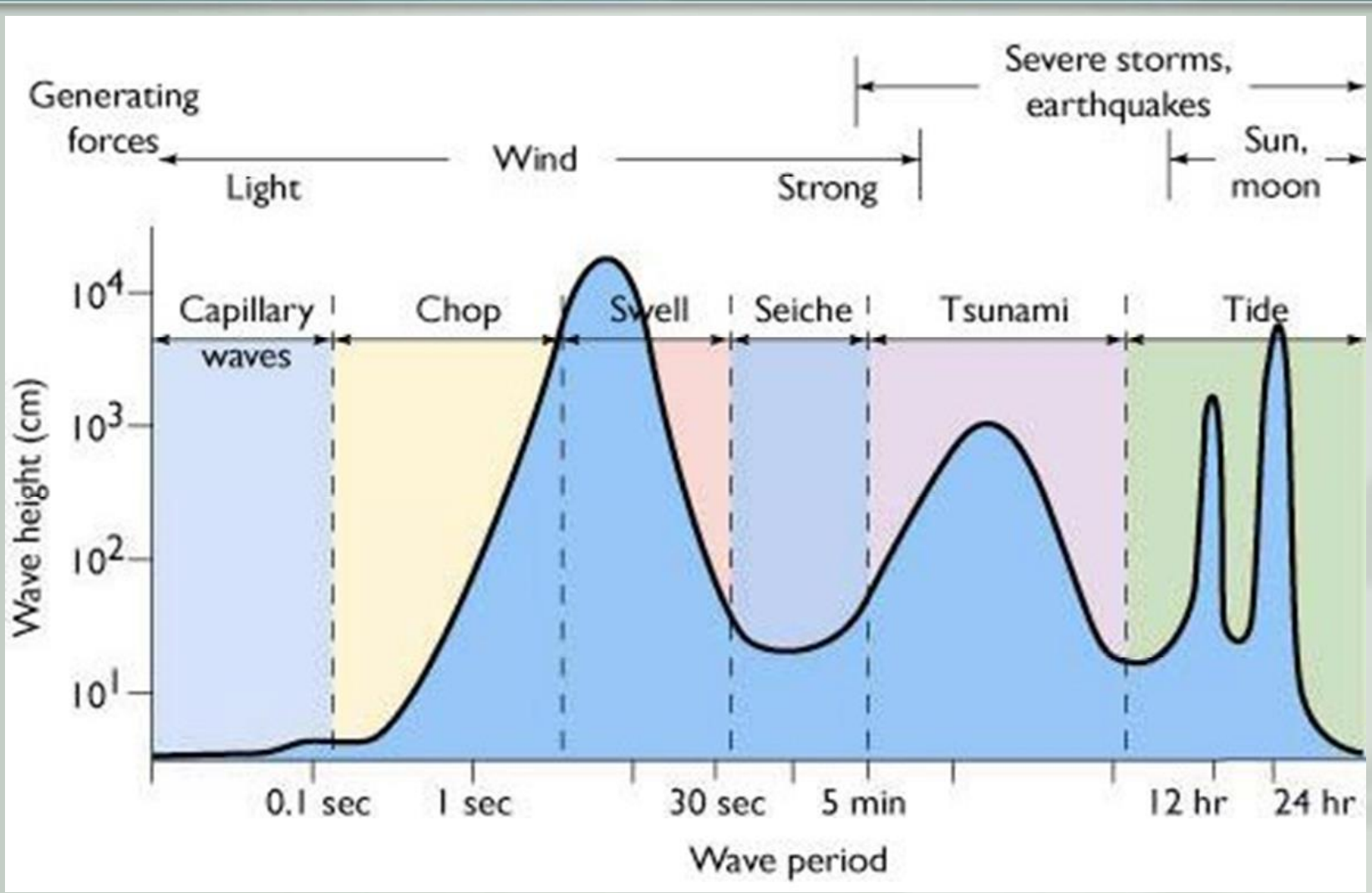
February 29, 2016

Types of Ocean Surface Waves



Waves measured by buoys

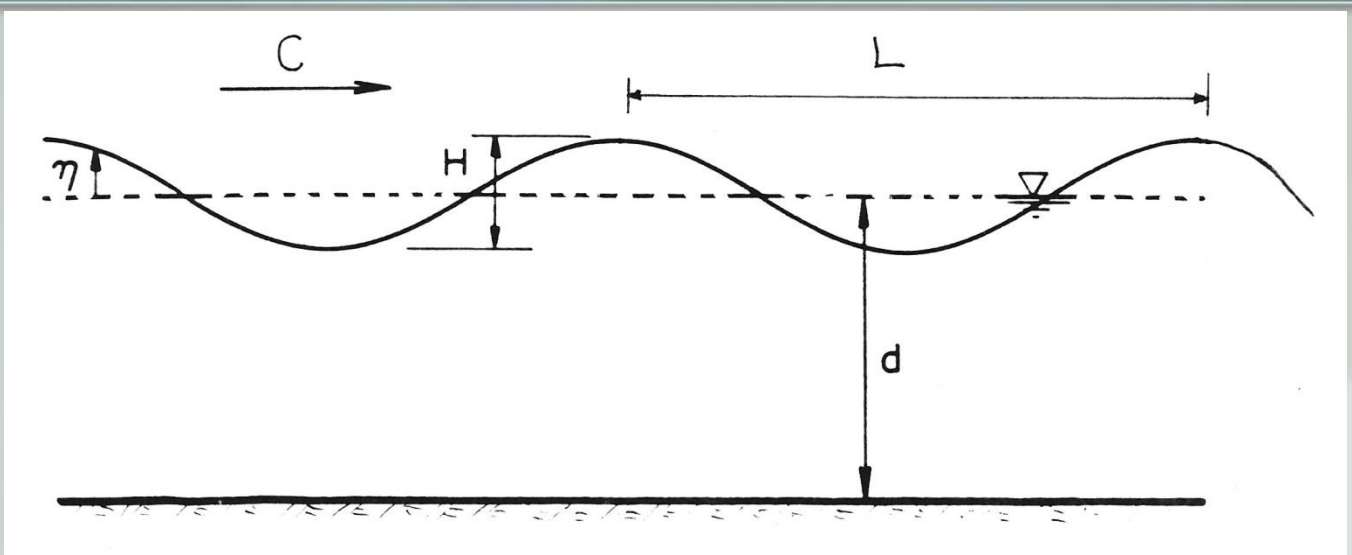
- Regular waves: most simple is small-amplitude or linear wave theory (Airy, 1845), which has many assumptions including sinusoidal waves in time and space, small amplitude, etc. (see Coastal Engineering Manual, 2008).
- Further assumptions for random waves:
  - Random (stochastic), Zero-Mean, Ergodic, Stationary, Gaussian (Ochi, 1998), Narrow banded spectrum (Tucker, 1991)
- Buoy Measurements make further assumptions:
  - Linearity, small angle approximations, simple harmonic oscillators (NDBC, 1996; Steele and Wang, 2004)
- Datawell buoys measure waves ~ 1.7 - 40 seconds and NDBC buoys measure waves ~ 2.0 – 33 seconds

Nondirectional and Directional Waves

- **Nondirectional waves** (also called point, one-dimensional, or omnidirectional waves) are vertical fluctuations of the water surface at a fixed location or point.

- **Directional waves** include information on both the nondirectional waves and directionality of the waves.

Linear Waves



- Water depth (d)
- Wave length (L)
- Wave height (H)
- Wave amplitude (a)
- Wave period (T)
- Wave steepness (H/L)
- Wave frequency ($f = 1/T$)
- Angular frequency ($\omega = 2\pi f = 2\pi/T$)
- Wave number ($k = 2\pi/L$)
- Wave speed ($C = L/T$)
- Wave profile $\eta = (H/2)\cos(kx-\omega t)$

Dispersion Relation

Dispersion relation relates the wave length of a wave to its frequency (or period). For a linear ocean wave,

$$\omega^2 = g \cdot \frac{2\pi}{L} \cdot tanh(\frac{2\pi d}{L})$$

$$C = \frac{L}{T} = \frac{gT}{2\pi} \cdot tanh(\frac{2\pi d}{L})$$

Deep water:

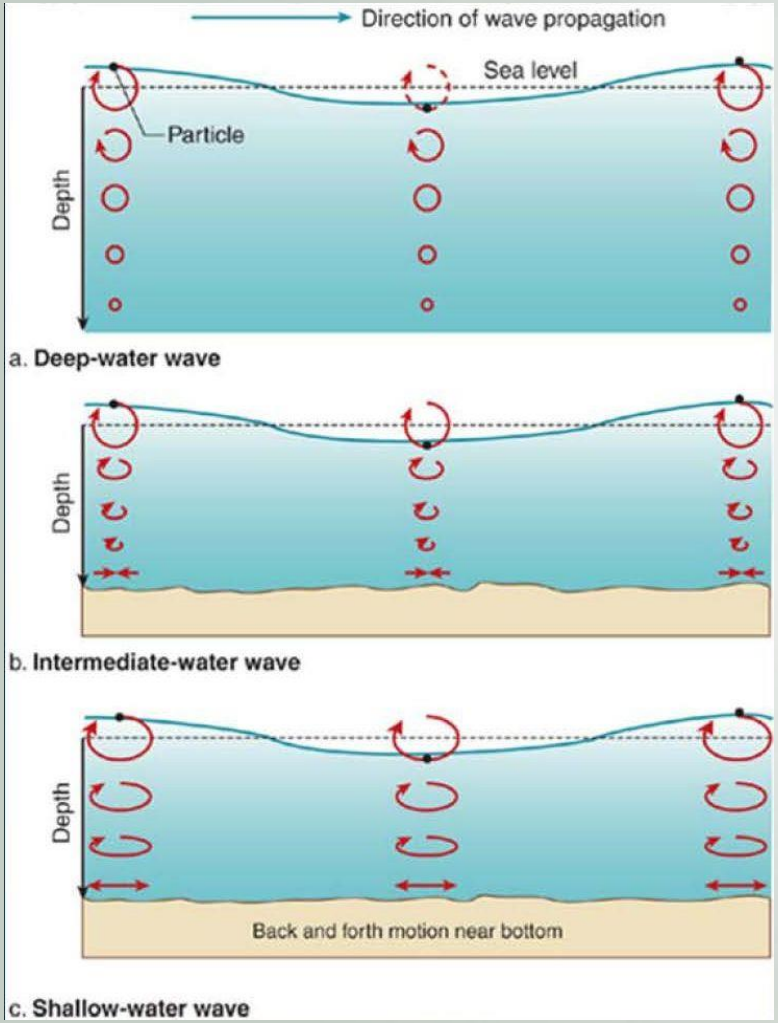
$C = \frac{g}{2\pi}T$ or $L = CT = \frac{g}{2\pi}T^2$

Shallow water:

$C = \sqrt{gd}$ or $L = CT = T\sqrt{gd}$

- Waves with different frequencies propagate at different velocities, i.e., long waves move faster than short waves.
- Waves consisting of various frequency components would separate or disperse due to the celerity difference.

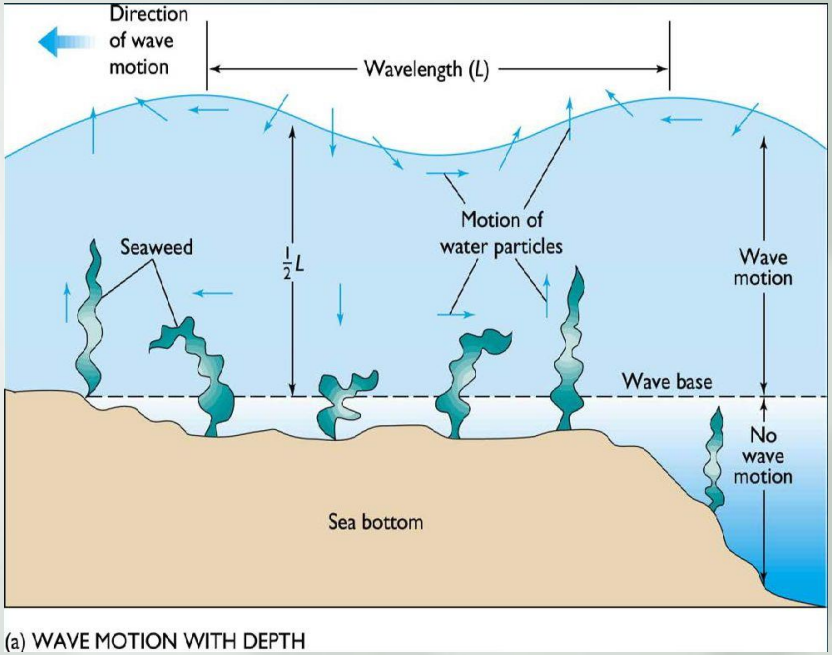
Particle orbital motion



$\frac{d}{L} > \frac{1}{2}$

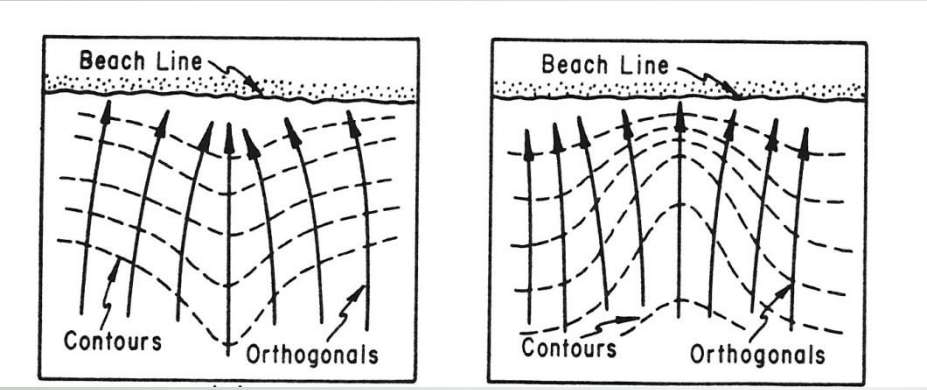
$\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$

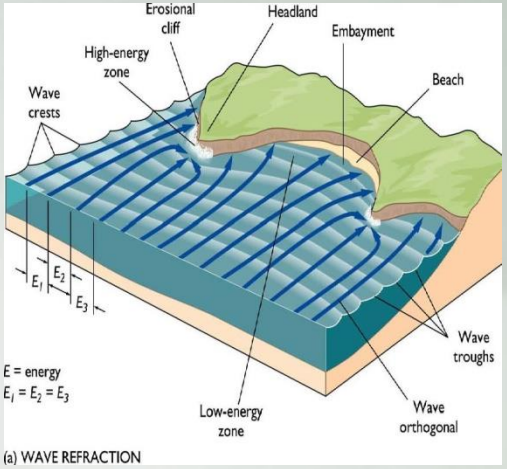
$\frac{d}{L} < \frac{1}{25}$



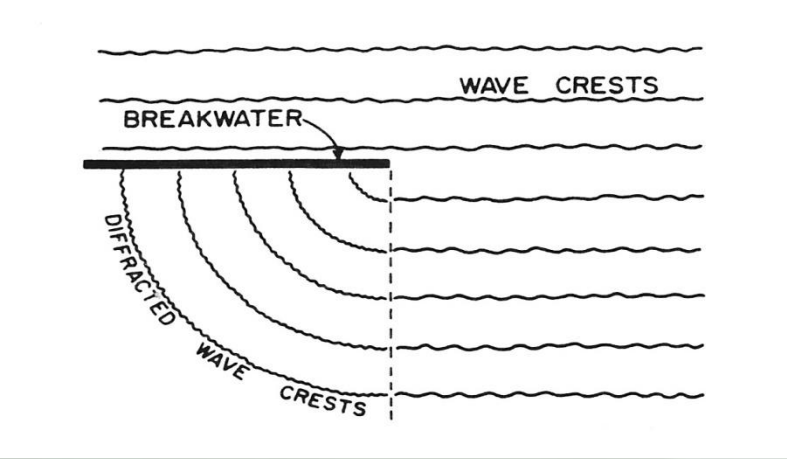
Wave refraction and diffraction

Wave Refraction



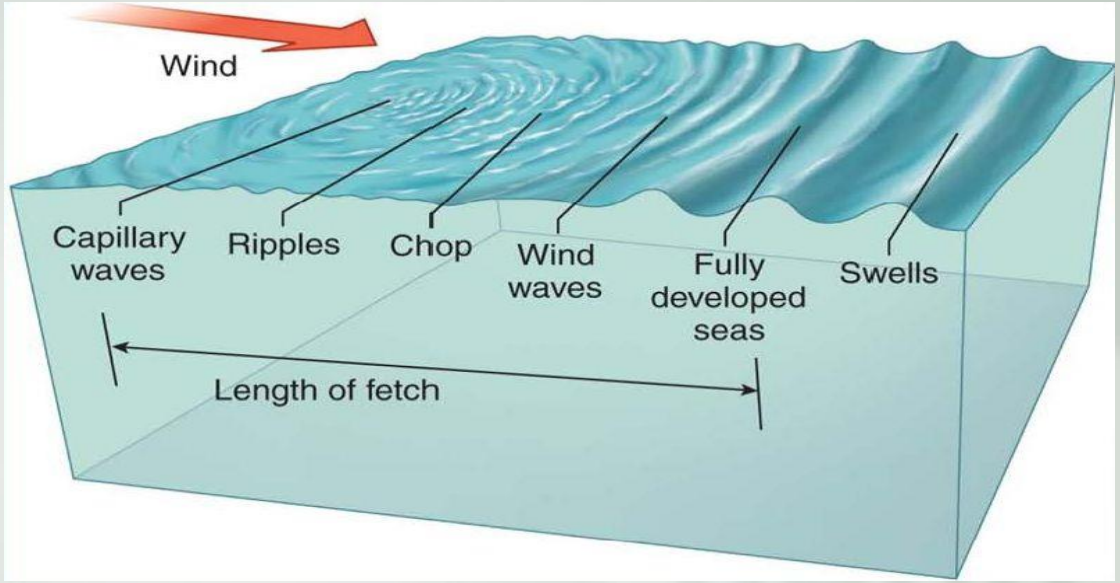


Wave Diffraction

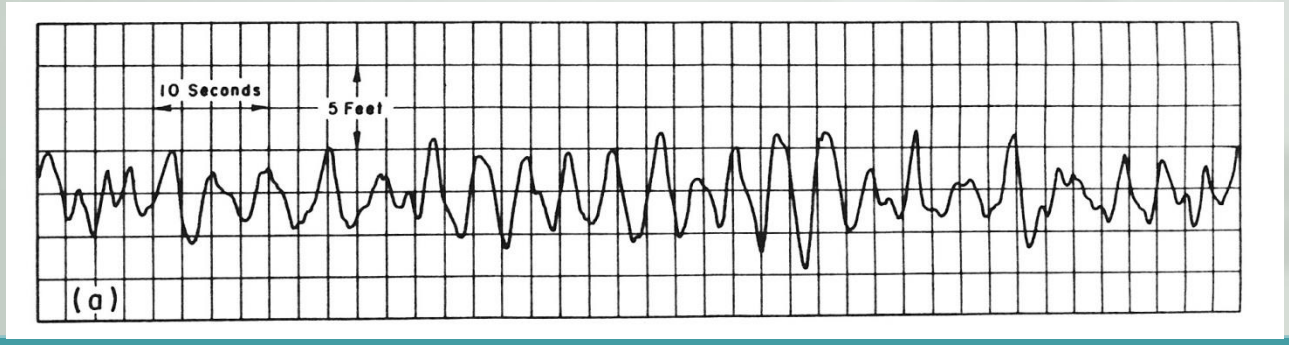


Wind waves and Random waves

Wind-Generated Waves

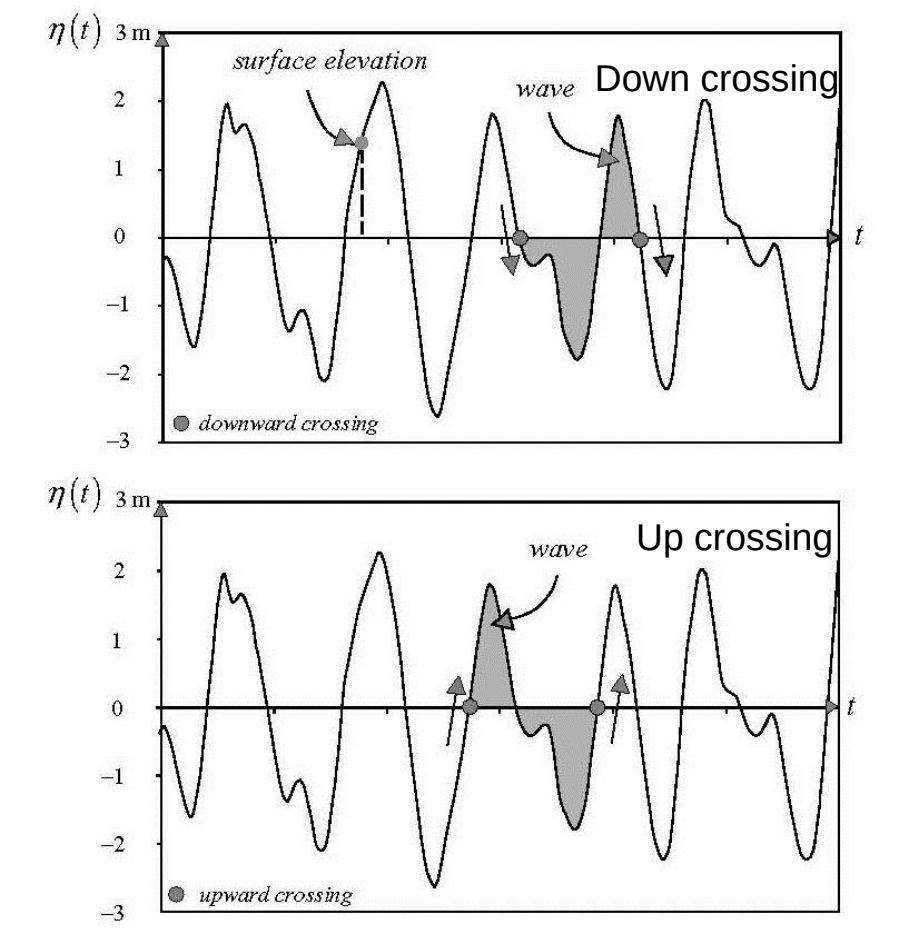


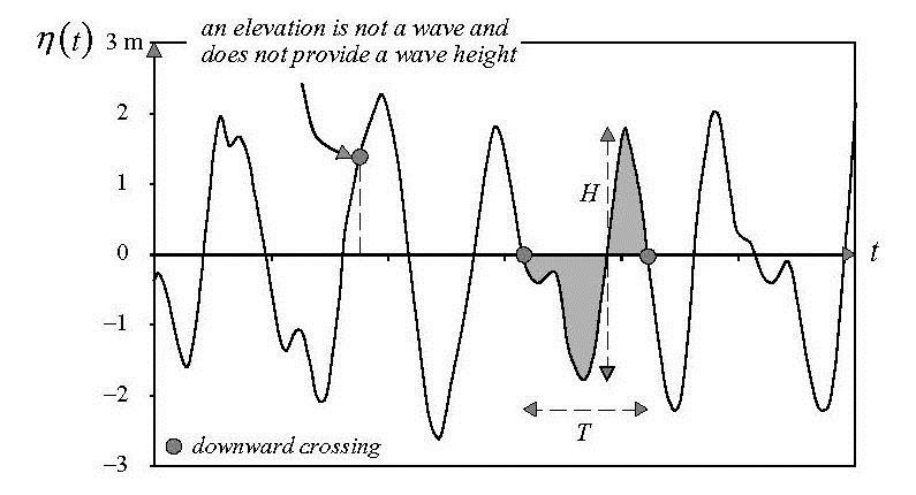
Random Waves



Wave data – Zero crossing analysis

Zero crossing analysis in time domain



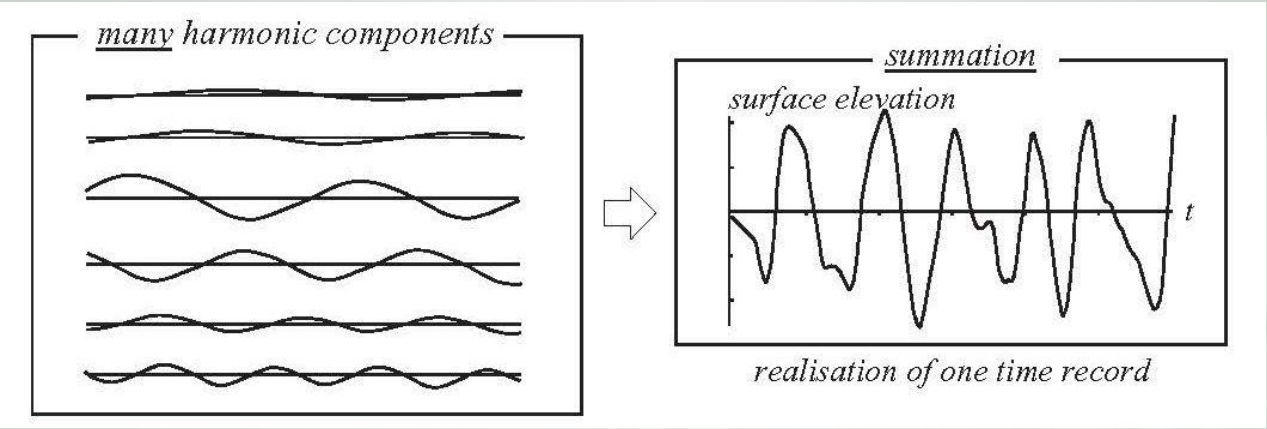


(from Holthuijsen, 2007)

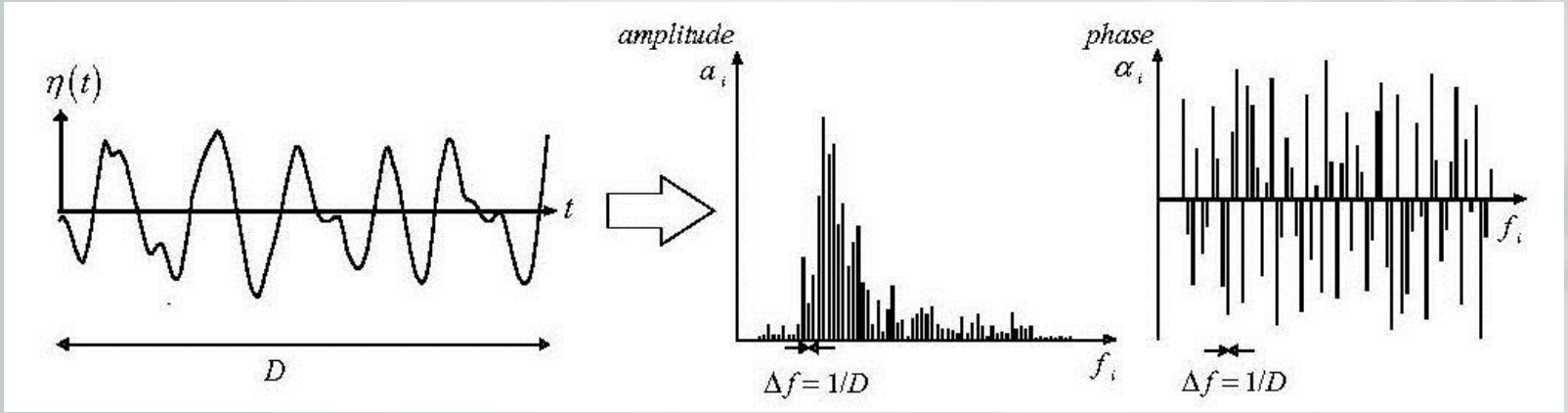
Wave data – Spectral analysis

Spectral analysis in frequency domain

Use Fourier analysis to convert a random wave record (in time domain) to a wave spectrum (in frequency domain)

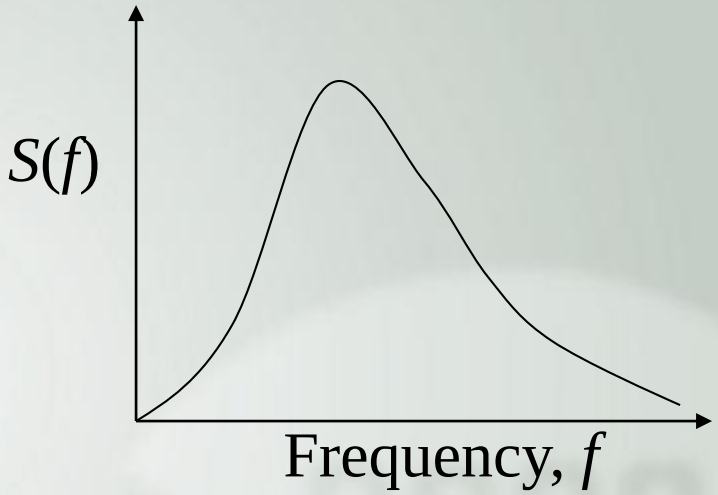


(from Holthuijsen, 2007)



Nondirectional wave data

- Wave energy spectrum



- Spectral Moments (m_n) used in calculating some of the bulk wave parameters – height and periods

$$m_n = \sum_{f(1)}^{f(n)} S(f) f^n \, \partial \mathrm{f}$$

Wave Heights

- Significant wave height (H_s) is the average wave height of the highest one-third of all the waves in a wave record ($H_{1/3}$).
  - In time domain: $H_{1/3}$ can be computed from a wave record using a zero-crossing analysis.
  - In frequency domain: if wave height distribution follows the Rayleigh distribution, the significant wave height is

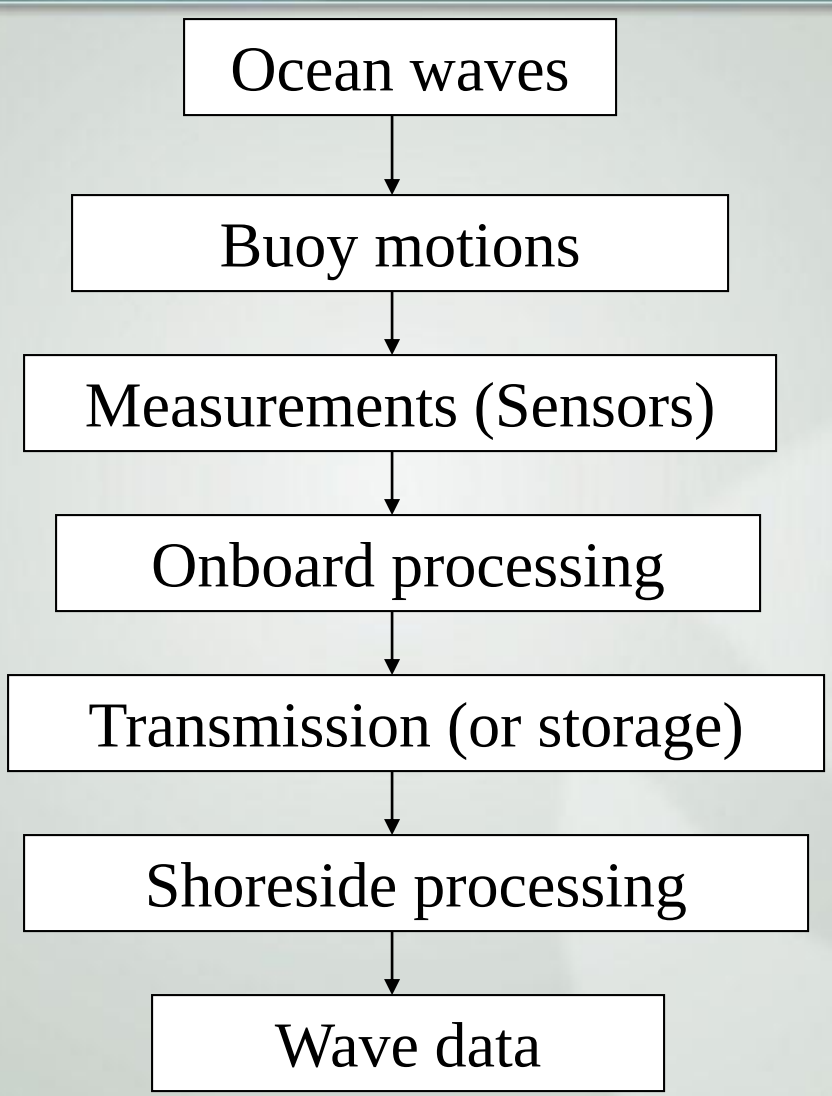
$$H_s \cong 4.005\sqrt{m_0}$$

- Maximum wave height (H_{max})
- Root-mean-square height (H_{rms})
- Average of tenth (hundredth) highest waves ($H_{1/10}$ or ($H_{1/100}$)

Wave Periods

- Dominant or Peak or Modal Period (T_p) – Inverse of frequency with maximum wave energy
- Average Periods (T_z or T_a) – there are a number of these used in various applications:
  - T_z = Average zero-crossing wave period; the time obtained by dividing the record length by the number of down-crossings (or up-crossings) in the record;
  - From Spectral Moments: $T_z = \sqrt{\frac{m_0}{m_2}}$

Buoy Wave Measurement



Nondirectional wave data from buoys

From buoy motion to wave data

$$S_w(f) = \frac{S_h(f)}{PTF}$$

$S_h(f)$: spectrum of buoy heave motion

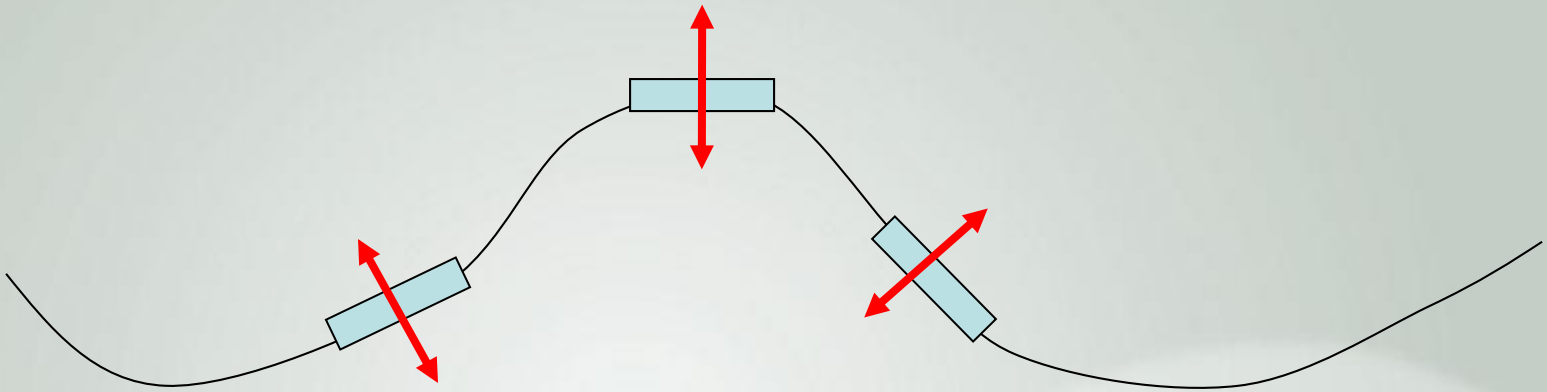
$S_w(f)$: wave spectrum (acceleration)

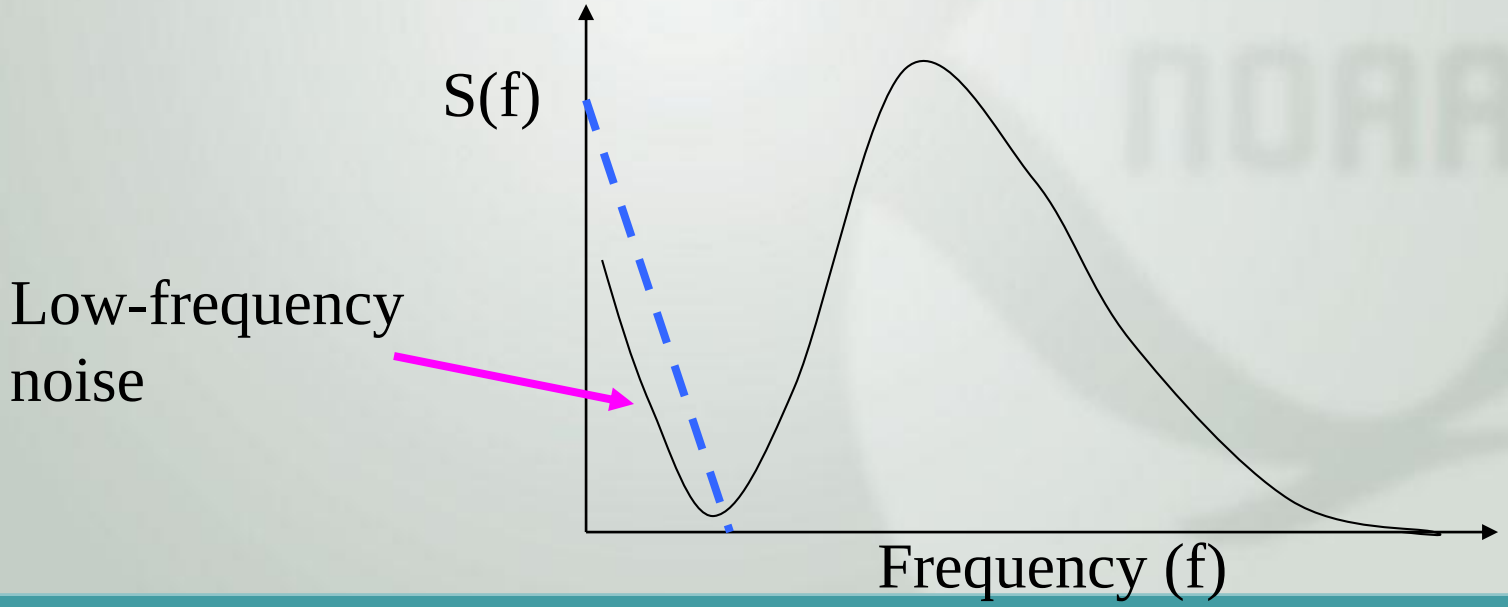
PTF: power transfer function

From acceleration to displacement spectrum

$$displacement\ spectrum = \frac{acceleration\ spectrum}{(2\pi f)^4}$$

Noise Correction in the Frequency Domain





Directional Random Ocean Waves

FIGURE 1.1 Aerial photo of sea waves.

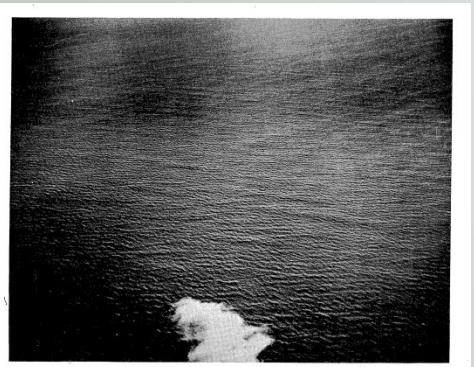
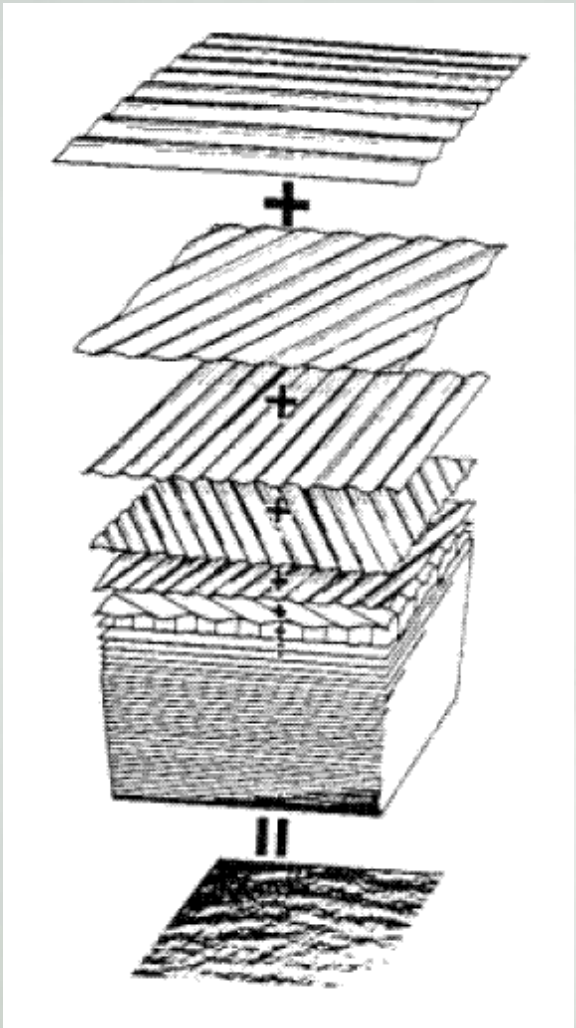


FIGURE 1.2 Aerial photo of swell waves.



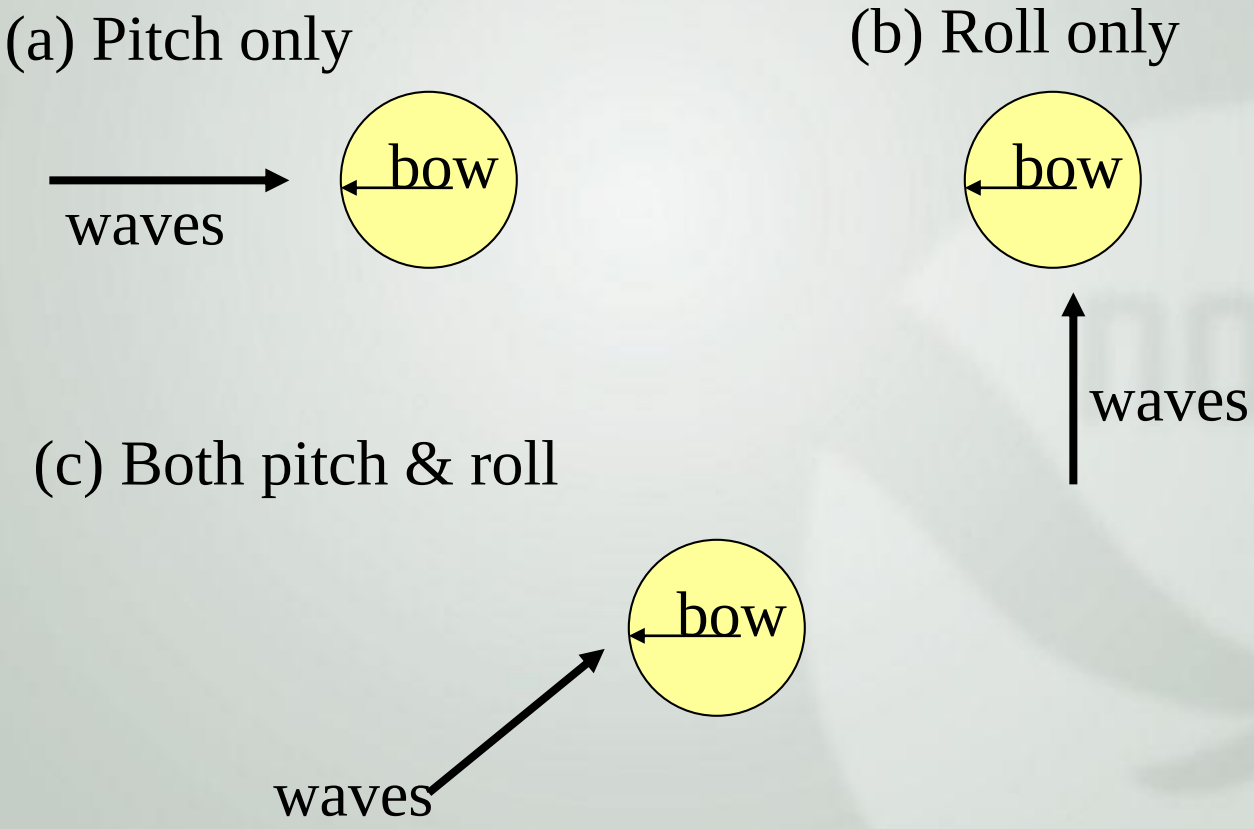
Pierson, W.J., G. Neumann and R.W. James, Hydrographic Office, 1955

Directional wave measurement from data buoys

- Surface-following (slope-following) principle
  - Water surface elevation and two orthogonal surface slopes (i.e., heave, pitch, and roll)
  - Longuet-Higgins *et al.,* 1963
- Particle-following (orbit-following) principle
  - Water surface elevation and two orthogonal horizontal particle velocities

Directional wave measurement from "pitch and roll" buoys

Based on "slope following" principle



Directional wave analysis (1)

$$S(f,\theta) = \frac{a_0}{2} + a_1 \cdot \cos\theta + b_1 \cdot \sin\theta + a_2 \cdot \cos 2\theta + b_2 \cdot \sin 2\theta + \cdots$$

θ = wave direction; f = wave frequency

a_n and b_n are the Fourier Coefficients

$$a_0 = \frac{1}{\pi} C_{11}; \quad a_1 = \frac{1}{\pi k} Q_{12}; \quad b_1 = \frac{1}{\pi k} Q_{13}$$

$$a_2 = \frac{1}{\pi k^2}\left(C_{22} - C_{33}\right); \quad b_2 = \frac{2}{\pi k^2} C_{23}$$

K: the wave number
C and *Q* : Co- and Quad-spectra

1: vertical motion (heave)
2: N-S slope (pitch)
3: W-E slope (roll)

Directional wave analysis (2)

$$S(f,\theta) = C_{11} \cdot \frac{1}{\pi}\left[\frac{1}{2} + r_1 \cos(\theta - \theta_1) + r_2 \cos 2(\theta - \theta_2)\right]$$

$$r_1 = \frac{1}{a_0}\sqrt{a_1^{\,2} + b_1^{\,2}}; \qquad r_2 = \frac{1}{a_0}\sqrt{a_2^{\,2} + b_2^{\,2}}$$

$$\theta_1 = \tan^{-1}(b_1, a_1); \qquad \theta_2 = \frac{1}{2}\tan^{-1}(b_2, a_2)$$

Transform to WMO/IAHR direction conventions:

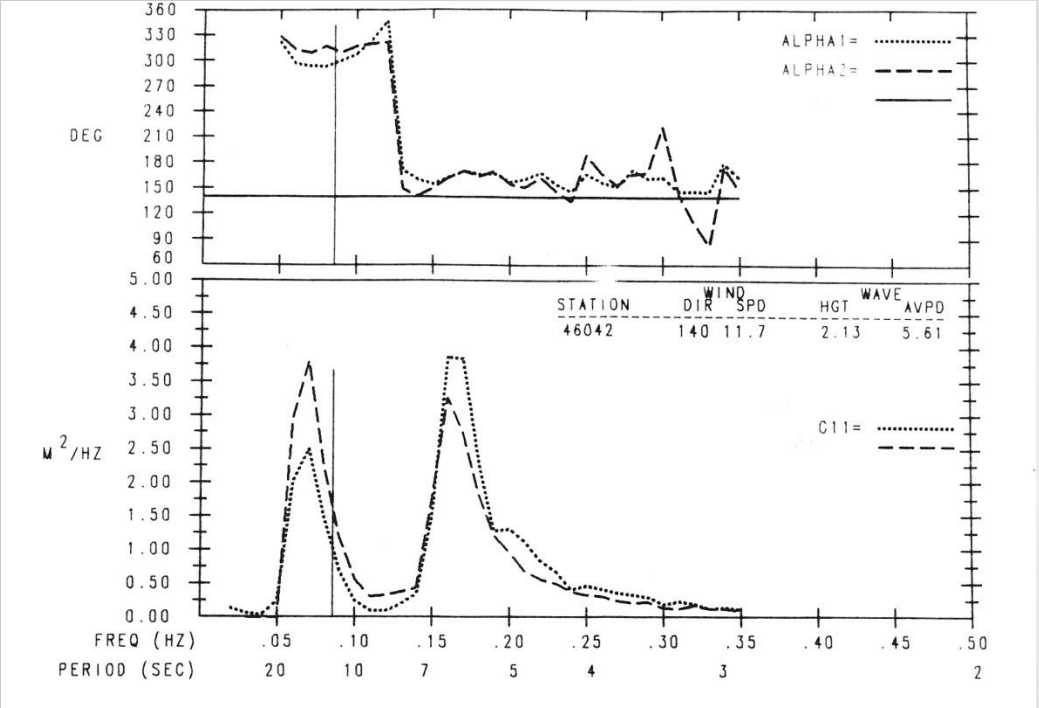
$$\alpha_1 = \frac{3\pi}{2} - \theta_1$$

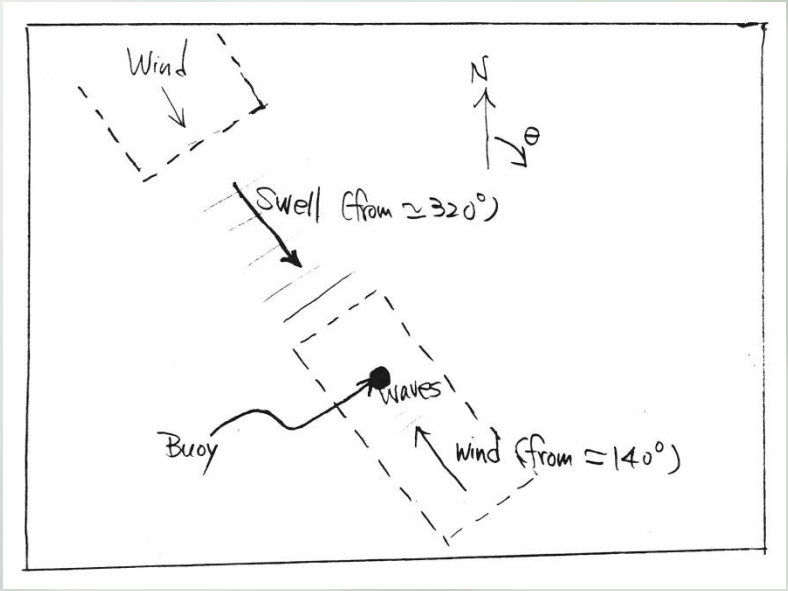
$$\alpha_2 = \frac{3\pi}{2} - \theta_2, \text{ +0 or } 180^\circ \text{ to minimize } |\alpha_1 - \alpha_2|$$

α_1 and α_2: Mean and Principal Wave Directions (direction FROM which waves originate, Positive clockwise, from East is 90°)

r_1 and r_2 : directional energy spreading

Directional wave example





Directional wave observations – “First-5” standard

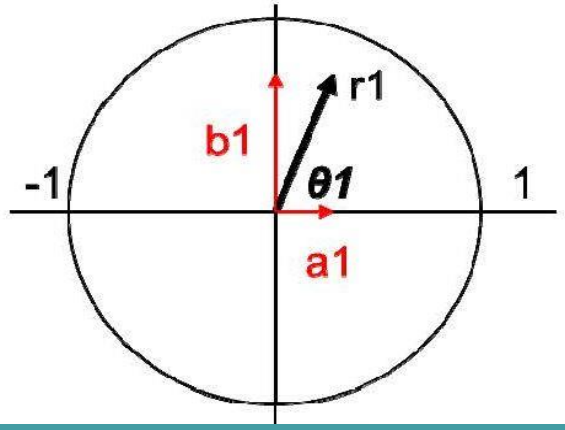
- mean direction
- directional spread
- skewness
- kurtosis

a1,b1,a2,b2

or, in NDBC format

- first-moment mean direction (*θ1*)
- first-moment spread parameter (*r1*)
- second-moment mean direction (*θ2*)
- second-moment spread parameter (*r2*)

C_{11} - wave energy



Thank you!

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References

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- Tucker, M.J. and E.G. Pitt, 2001, "*Waves in Ocean Engineering"*, Elsevier, 521 pp.
- Holthuijsen, L.H., 2007, "Waves in Oceanic and Coastal Waters", Cambridge, 379 pp.
- WMO, 1998*: Guide to Wave Analysis and Forecasting*.

Resources

COMET® and MetEd Programs

Spanish Versions also have French versions

Wave Life Cycle I: Generation

http://www.meted.ucar.edu/marine/mod2_wlc_gen/index.htm

Ciclo de vida de las olas I: Generación

http://www.meted.ucar.edu/marine/mod2_wlc_gen_es/index.htm

Wave Life Cycle II: Propagation & Dispersion

http://www.meted.ucar.edu/marine/mod3_wlc_propdis/index.htm

Ciclo de vida de las olas II: propagación y dispersión

http://www.meted.ucar.edu/marine/mod3_wlc_propdis_es/index.htm

Wave Types and Characteristics

http://www.meted.ucar.edu/marine/mod1_wv_type_char/index.htm

Tipos de olas y sus características

https://www.meted.ucar.edu/training_module_es.php?id=1017

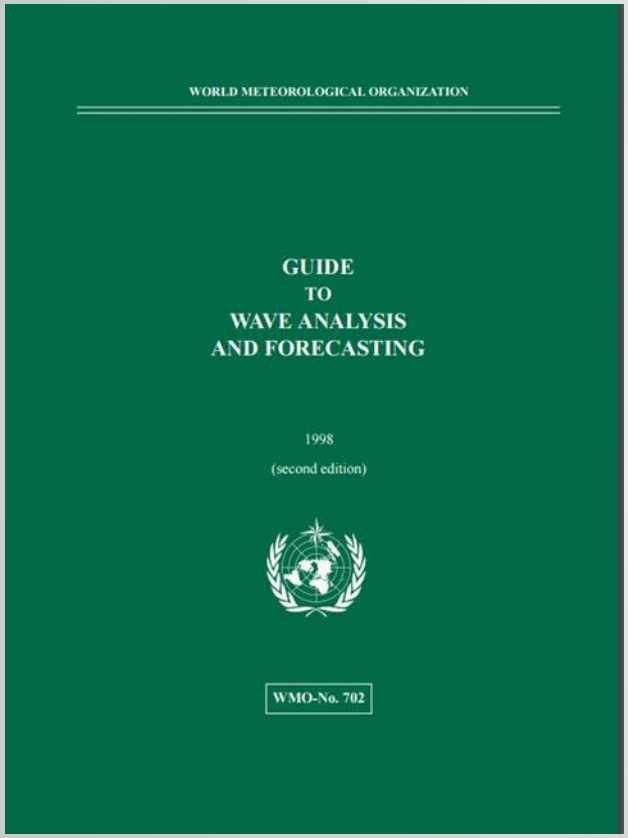
Analyzing Ocean Swell

http://www.meted.ucar.edu/oceans/ocean_swell

Resources

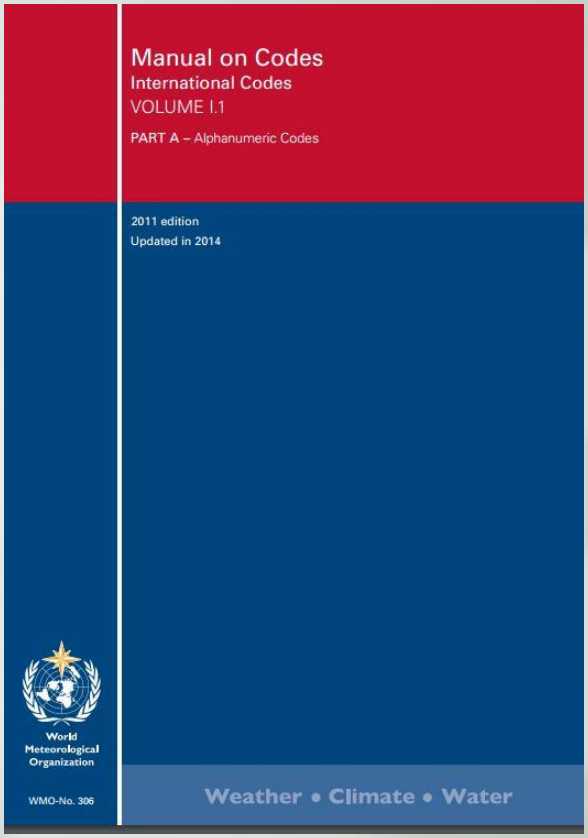
World Meteorological Organization (WMO)

WMO, 1998. Guide to Wave Analysis and Forecasting



https://www.wmo.int/pages/prog/amp/mmop/documents/WMO%20No%20702/WMO702.pdf

WMO, 2014. Manual On Codes, see WAVEOB

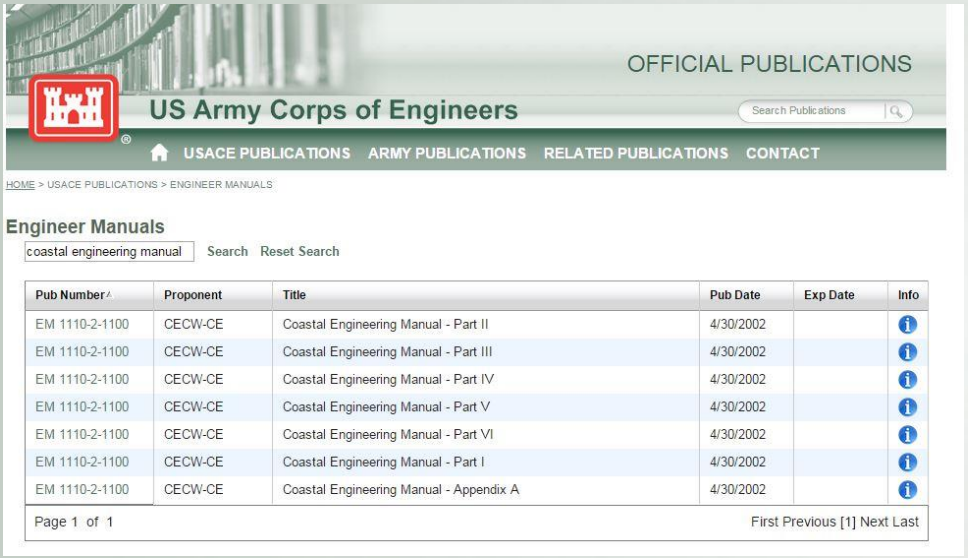


http://www.wmo.int/pages/prog/www/WMOCodes/WMO306_vI1/VolumeI.1.html

Resources

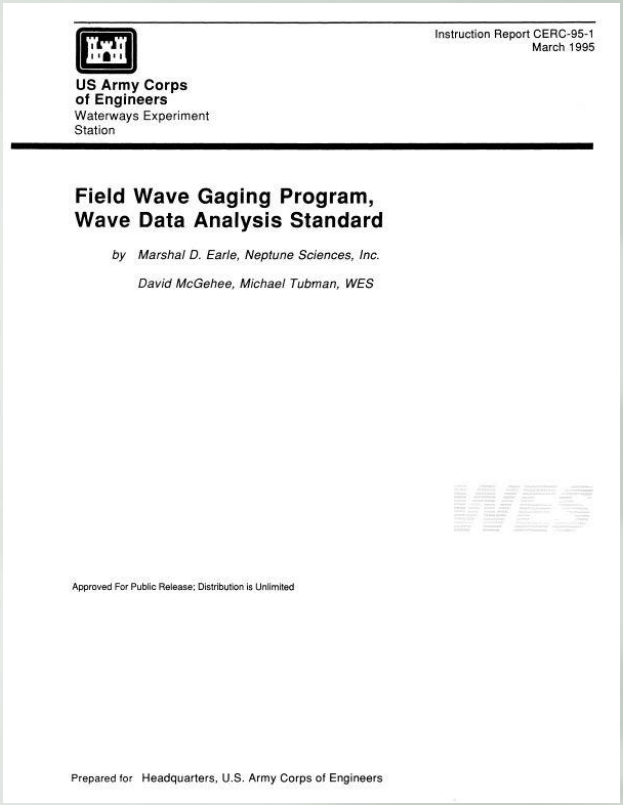
U.S. Army Corps of Engineers

Coastal Engineering Manual



http://www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1100_Part-01.pdf

US Army Corps of Engineers, 1995. Field Wave Gaging Program.

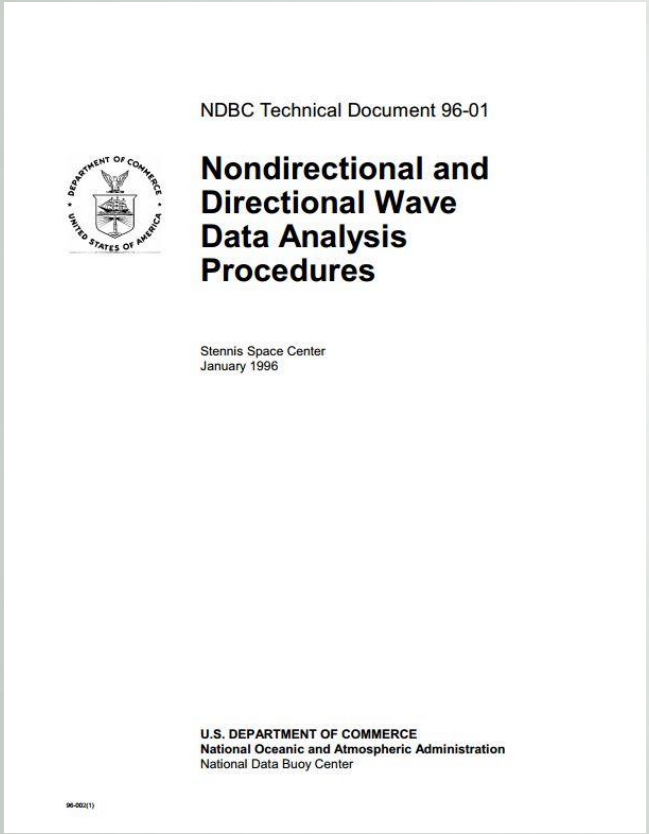


http://cdip.ucsd.edu/SCBPS/DOCS/Publications/Article/USACE.pdf

Resources

NDBC, 1996. Wave Data Analysis

(Undergoing revision)



NDBC Technical Document 96-01

Nondirectional and Directional Wave Data Analysis Procedures

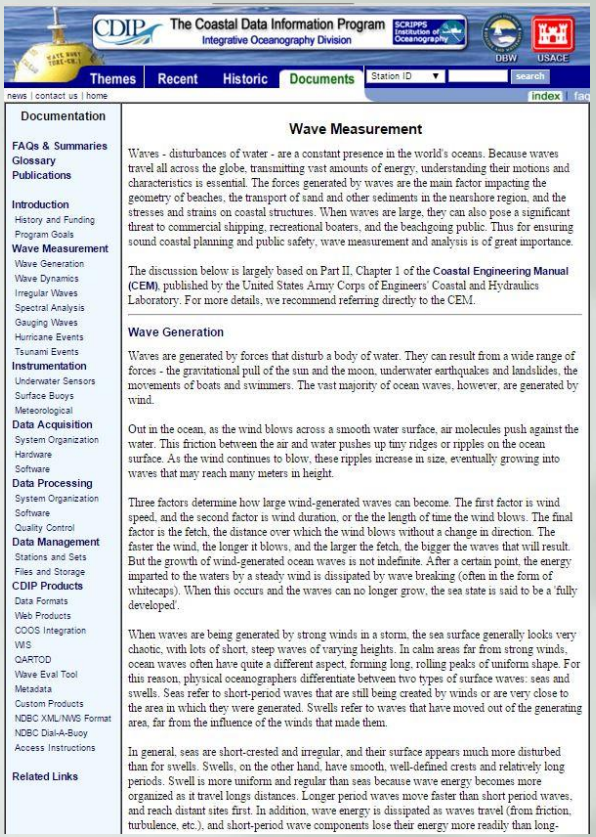
Stennis Space Center
January 1996

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Data Buoy Center

96-002(1)

http://www.ndbc.noaa.gov/wavemeas.pdf

CDIP Documentation



CDIP The Coastal Data Information Program
Integrative Oceanography Division
SCRIPPS Institution of Oceanography
DBW USACE

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Documentation

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Wave Measurement

Waves - disturbances of water - are a constant presence in the world's oceans. Because waves travel all across the globe, transmitting vast amounts of energy, understanding their motions and characteristics is essential. The forces generated by waves are the main factor impacting the geometry of beaches, the transport of sand and other sediments in the nearshore region, and the stresses and strains on coastal structures. When waves are large, they can also pose a significant threat to commercial shipping, recreational boaters, and the beachgoing public. Thus for ensuring sound coastal planning and public safety, wave measurement and analysis is of great importance.

The discussion below is largely based on Part II, Chapter 1 of the **Coastal Engineering Manual (CEM)**, published by the United States Army Corps of Engineers' Coastal and Hydraulics Laboratory. For more details, we recommend referring directly to the CEM.

Wave Generation

Waves are generated by forces that disturb a body of water. They can result from a wide range of forces - the gravitational pull of the sun and the moon, underwater earthquakes and landslides, the movements of boats and swimmers. The vast majority of ocean waves, however, are generated by wind.

Out in the ocean, as the wind blows across a smooth water surface, air molecules push against the water. This friction between the air and water pushes up tiny ridges or ripples on the ocean surface. As the wind continues to blow, these ripples increase in size, eventually growing into waves that may reach many meters in height.

Three factors determine how large wind-generated waves can become. The first factor is wind speed, and the second factor is wind duration, or the the length of time the wind blows. The final factor is the fetch, the distance over which the wind blows without a change in direction. The faster the wind, the longer it blows, and the larger the fetch, the bigger the waves that will result. But the growth of wind-generated ocean waves is not indefinite. After a certain point, the energy imparted to the waters by a steady wind is dissipated by wave breaking (often in the form of whitecaps). When this occurs and the waves can no longer grow, the sea state is said to be a 'fully developed'.

When waves are being generated by strong winds in a storm, the sea surface generally looks very chaotic, with lots of short, steep waves of varying heights. In calm areas far from strong winds, ocean waves often have quite a different aspect, forming long, rolling peaks of uniform shape. For this reason, physical oceanographers differentiate between two types of surface waves: seas and swells. Seas refer to short-period waves that are still being created by winds or are very close to the area in which they were generated. Swells refer to waves that have moved out of the generating area, far from the influence of the winds that made them.

In general, seas are short-crested and irregular, and their surface appears much more disturbed than for swells. Swells, on the other hand, have smooth, well-defined crests and relatively long periods. Swell is more uniform and regular than seas because wave energy becomes more organized as it travel longs distances. Longer period waves move faster than short period waves, and reach distant sites first. In addition, wave energy is dissipated as waves travel (from friction, turbulence, etc.), and short-period wave components lose their energy more readily than long-

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IAHR/PIANC, 1987. List of Sea-State Parameters

Journal of Waterway, Port, Coastal, and Ocean Engineering

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Abstract | References | Complete PDF (770 KB) | Cited By

The IAHR Working Group on Wave Generation and Analysis (1989). "List of Sea-State Parameters." *J. Waterway, Port, Coastal, Ocean Eng.*, 10.1061/(ASCE)0733-950X(1989)115:6(793), 793-808.

TECHNICAL PAPERS

List of Sea-State Parameters

The IAHR Working Group on Wave Generation and Analysis

[1]*Int. Assoc. for Hydraulic Res., IAHR Secretariat, P.O. Box 177, 2600 MH, Delft, The Netherlands*

A comprehensive list of wave parameters and wave-related functions is developed to accommodate modern laboratory and field programs and to help investigators clarify symbols, nomenclature, and definitions used. The list is suggested as a reference with the aim of promoting uniformity and reducing the confusion of present usage. Parameters in the list are grouped for easier reference. The list was originally published by the International Association for Hydraulic Research (IAHR) and Permanent International Association of Navigation Congresses (PIANC). It is provided here to increase its accessibility and use by ASCE members, readers, and contributors.

Article History

Published: 01 November 1989

Publication Data

ISSN (print): 0733-950X

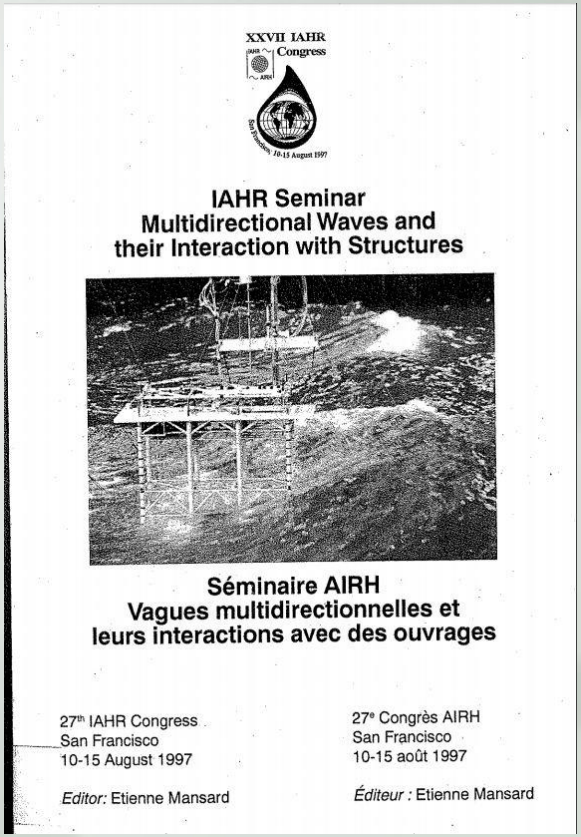
ISSN (online): 1943-5460

Publisher: American Society of Civil Engineers

Permalink: http://dx.doi.org/10.1061/(ASCE)0733-950X(1989)115:6(793)

ASCE Subject Headings: Wave height, Waves, Wave spectra, Parameters, Standardization, Terminology

IAHR, 1997. List of Sea State Parameters – Update for multidirectional waves



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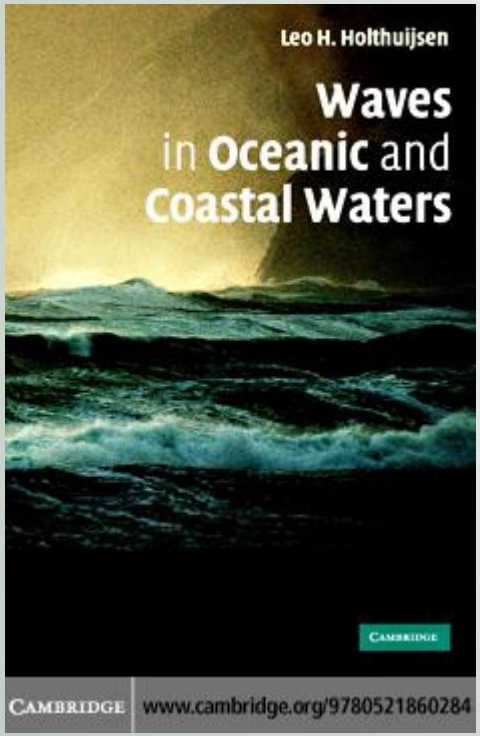
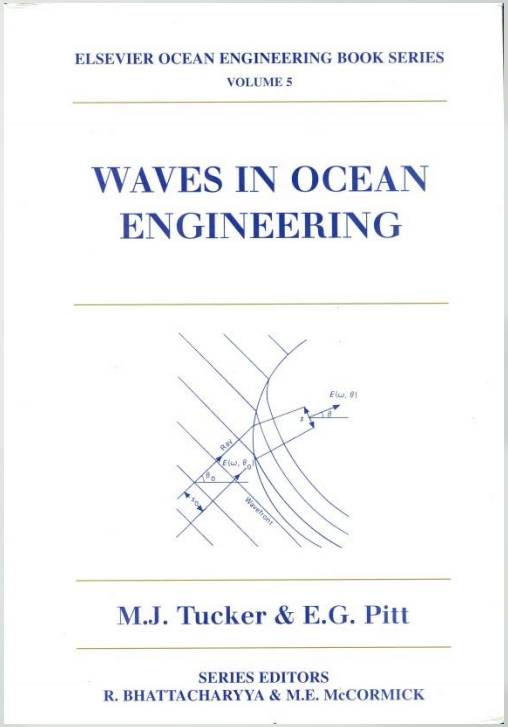
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